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On the Possible Role of the Stellar Drift Motions for the Dynamics and Structure of Differentially Rotating Stellar Systems

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Abstract

The differential rotation of stellar systems leads to the appearance of specific drift motions of stars. These motions may cause the excitation of spiral waves. This possibility is investigated locally in the present paper. We conclude that drift motions may be of importance for generating the spiral structure of galaxies, especially of galaxies of the Sc type.

I. Introduction

The density wave theory has been rapidly developed recently in Stellar Dynamics. To a great extent, this development was stimulated by the resemblance between a rarefied plasma and collisionless self-gravitating systems. For this reason the methods used in plasma physics are widely used in stellar systems also.

Initially, the density wave theory was applied to purely dynamical problems – stability and collisionless evolution problems (SIMON, 1961; LYNDEN-BELL, 1962; SWEET, 1963; TOOMRE, 1964; LEBEDEV *et al.*, 1965; MAKSUMOV and MAROCHNIK, 1965a, b; LEE, 1967; MAROCHNIK and PTITSINA, 1968; WU, 1968; SUCHKOV, 1969; BISNOVATY-KOGAN *et al.*, 1969). LIN (LIN and SHU, 1964, 1966; LIN *et al.*, 1969) and many others (BECKER and CONTOPOULOS, 1970) applied the density wave theory to the problems of spiral structure of galaxies.

But the wave theory of spiral structure encounters difficulties due to the antispiral theorem (LYNDEN-BELL and OSTRIKER, 1967) and the need of an energy source in order to maintain the spiral pattern (TOOMRE, 1968). Both these difficulties can be overcome, probably, if there is an effective instability (overstability).

One attempt to avoid the above difficulties, is based on the interaction between the flat and spherical subsystems of spiral galaxies (MAROCHNIK and SUCHKOV, 1969). This interaction is supposed to be responsible for a beam-type overstability, similar to that of plasma physics. This mechanism implies that a considerable part of the rotation energy of the flat subsystem can be transformed into density wave energy. But in fact this is not quite correct, because in such a case the flat subsystem would be scattered by the spherical one very fast. The energy of density fluctuations arising from the flat system cannot exceed the energy of its random stellar motions. This energy is two orders of magnitude less than the rotational energy. Perhaps the interaction between spherical and flat subsystems can maintain the tightly wound spiral structure of Sa and Sb galaxies.

But this is impossible in the case of Sc galaxies, where the spherical subsystem is rather weak.

However there is another energy source which is comparable with the energy of the random motions, and even more powerful. This is the differential rotation. The differential rotation energy of the whole galactic disk is only one order of magnitude less than the total rotation energy.

As it will be shown below, some dynamical effects are connected with the kinematic and spatial inhomogeneity of stellar systems. These effects are due to the drift motions of stars. As in the case of phenomena studied in plasma physics (TCERKOVNIKOV, 1957; RUDAKOV and SAGDEEV, 1959, 1961; VEDENOV *et al.*, 1961; ROSENBLUTH *et al.*, 1962; KADOMTSEV and TIMOFEEV, 1962; GALEEV *et al.*, 1963a; MIKHAILOVSKY, 1963; MIKHAILOVSKAYA and MIKHAILOVSKY, 1963; RUKHADZE and SILIN, 1964), these effects are expressed as a specific drift overstability of the density waves. This overstability arises because the drift motions of stars excite density waves coherently. Drift overstabilities are of great importance for the structure and dynamics of stellar systems, in particular for the formation and maintenance of spiral waves in galaxies. They are considered in detail elsewhere (MAKSUMOV, 1973a, b, c; MAKSUMOV and MISHUROV, 1973), but their results are summarized here. A similar opinion was expressed by LYNDEN-BELL (1970), without a quantitative analysis.

According to rough quasilinear estimates (MAKSUMOV, 1973b, c) almost all the energy of differential rotation may be transformed into energy of density waves.

A simple axisymmetric model will be considered, namely a thin stellar disk. Boundary conditions are not taken into account because the problem is a local one. The treatment is based on the collisionless kinetic equation because we deal with either a collisionless gas of "clouds" or a collisionless gas of stars. Moreover, drift instabilities are a specific "kinetic" effect.

II. Differentially Rotating Disk Galaxy

Consider a differentially rotating infinitely thin inhomogeneous stellar disk. In configuration space introduce a cylindrical coordinate system (ρ, φ, z), z being the rotation axis. Separate the circular rotational velocity of stars and introduce residual velocity variables as follows

$$v_\perp = (v_\rho^2 + v_\varphi^2)^{\frac{1}{2}}, \qquad \alpha = \tan^{-1}(v_\varphi / v_\rho),$$

where v_ρ and v_φ are projections of the residual velocity on the radial coordinate and perpendicularly to it. The angular velocity of rotation $\Omega = \Omega(\rho)$ is a function of the ρ variable only. In these variables, the kinetic equation takes the form

$$\begin{aligned}
&\frac{\partial f}{\partial t} + v_\perp \cos\alpha \frac{\partial f}{\partial \rho} + \left(\frac{v_\perp \sin\alpha}{\rho} + \Omega\right) \frac{\partial f}{\partial \varphi} \\
&+ \left(\left[\frac{\partial \psi}{\partial \rho} + \Omega^2 \rho\right] \cos\alpha + \frac{\sin\alpha}{\rho} \frac{\partial \psi}{\partial \varphi} - \rho \frac{d\Omega}{d\rho} v_\perp \cos\alpha \sin\alpha\right) \frac{\partial f}{\partial v_\perp} \qquad (1) \\
&+ \left(-\left[\frac{\partial \psi}{\partial \rho} + \Omega^2 \rho\right] \frac{\sin\alpha}{v_\perp} + \frac{\cos\alpha}{v_\perp \rho} \frac{\partial \psi}{\partial \varphi} - \frac{v_\perp \sin\alpha}{\rho} - \rho \frac{d\Omega}{d\rho} \cos^2\alpha\right) \frac{\partial f}{\partial \alpha} - 2\Omega \frac{\partial f}{\partial \alpha} = 0.
\end{aligned}$$

The distribution function $f(\rho, \varphi, v_\perp, \alpha, t)$ is normalized to the surface number density of stars. $\psi(\rho, \varphi, t)$ is the gravitational potential of the system.

In the theory of tightly wound spiral arms, the inclination angle i is assumed to be extremely small $\left(\tan i = \frac{m}{k_\rho \rho}\right.$, where m/ρ and k_ρ are the azimuthal and radial wave numbers of a spiral mode). If i is small, the drift effects may be ignored. In fact, the ratio of the third term of the kinetic equation (1)

$$\left(\frac{v_\varphi}{\rho}\frac{\partial f}{\partial \varphi} = \frac{v_\perp \sin\alpha}{\rho}\frac{\partial f}{\partial \varphi}\right),$$

which describes the drift, to the second one

$$\left(v_\rho \frac{\partial f}{\partial \rho} = v_\perp \cos\alpha \frac{\partial f}{\partial \rho}\right)$$

is just of the order of $\tan i$. But it is clear that the magnitude of the inclination angle of real spirals is finite. Generally speaking, this fact cannot be ignored.

The kinetic equation which describes the slow, in comparison with the epicyclic motion, processes, may be obtained by averaging of Eq. (1) over the angle α. Approximately, this angle may be considered as varying uniformly in time.

Writing out the characteristics of the Eq. (1) and omitting the part of the gravitational potential which is balanced by rotation, we find by expansion in inverse powers of (2Ω) and subsequent averaging over the angle α, the following averaged characteristics (BOGOLYUBOV and ZUBAREV, 1955; BOGOLYUBOV and MITROPOLSKY, 1963; RUDAKOV and SAGDEEV, 1958):

$$\begin{aligned} \dot{\bar{\rho}} &= \frac{1}{\kappa\rho}\frac{\partial\psi}{\partial\bar{\varphi}}, \qquad \dot{\bar{\varphi}} = \Omega + \frac{1}{4}\frac{\bar{v}_\perp^2}{\kappa^2\bar{\rho}}\frac{d\Omega}{d\bar{\rho}} - \frac{1}{\kappa\bar{\rho}}\frac{\partial\psi}{\partial\bar{\rho}}, \\ \dot{\bar{v}}_\perp &= \frac{1}{4}\frac{\bar{v}_\perp}{\kappa^2\bar{\rho}}\frac{d\Omega}{d\bar{\rho}}\frac{\partial\psi}{\partial\bar{\varphi}}, \qquad \kappa = 2\Omega + \frac{1}{2}\bar{\rho}\frac{d\Omega}{d\bar{\rho}}. \end{aligned} \tag{2}$$

The bar here denotes averaging, and it will be suppressed below. It is seen that there are two types of drift, one due to differential rotation and another one due to the orthogonality between the angular velocity and the gravitational force.

In averaged variables $(\rho, \varphi, v_\perp)$ the kinetic equation for $f(\rho, \varphi, v_\perp, t)$, as a continuity equation, has the form

$$\frac{\partial f}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\,\dot{\rho}\,f) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}(\rho\,\dot{\varphi}\,f) + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}(v_\perp\,\dot{v}_\perp\,f) = 0. \tag{3}$$

where $\dot{\rho}$, $\dot{\varphi}$ and $\dot{v}_\perp$ are defined by Eq. (2).

Note that the kinetic equation may be written in the form

$$\frac{\partial f}{\partial t} + \dot{\rho}\frac{\partial f}{\partial\rho} + \dot{\varphi}\frac{\partial f}{\partial\varphi} + \dot{v}_\perp\frac{\partial f}{\partial v_\perp} = 0. \tag{4}$$

Of course, (3) and (4) are equivalent. This requires

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\,\dot{\rho}) + \frac{1}{\rho}\frac{\partial}{\partial\varphi}(\rho\,\dot{\varphi}) + \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}(v_\perp\,\dot{v}_\perp) = 0. \tag{5}$$

Thus Eq. (2) must satisfy the relation (5). Substituting (2) into (5) we see that it is satisfied in our approximation in which by differentiation with respect to ρ, (i) only $\left(1+\frac{1}{4}\rho\,\Omega^{-1}\frac{d\Omega}{d\rho}\right)$ in the expression for κ must be differentiated, because (2Ω) is assumed to be a fixed parameter, in obtaining the relations (2), and (ii) the second derivative of Ω with respect to ρ and the second power of the derivative of Ω with respect to ρ must be neglected.

After this remark substitute the relations (2) into Eq. (3). Then we find the kinetic equation needed

$$\frac{\partial f}{\partial t}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{1}{\kappa}\frac{\partial\psi}{\partial\varphi}f\right)+\frac{1}{\rho}\frac{\partial}{\partial\varphi}\left[\rho\left(\Omega+\frac{1}{4}\frac{v_\perp^2}{\kappa^2\rho}\frac{d\Omega}{d\rho}-\frac{1}{\kappa\rho}\frac{\partial\psi}{\partial\rho}\right)f\right]+\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(\frac{v_\perp^2}{4\kappa^2\rho}\frac{d\Omega}{d\rho}\frac{\partial\psi}{\partial\varphi}f\right)=0. \tag{6}$$

Now we linearize Eq. (6) assuming that the stationary state is axisymmetric and the gravitational force is balanced by the centrifugal force. Then

$$\frac{\partial f_0}{\partial t}=\frac{\partial f_0}{\partial\varphi}=\frac{\partial\psi_0}{\partial\varphi}=\frac{\partial\psi_0}{\partial\rho}=0,$$

where f_0 and ψ_0 are the distribution function and the gravitational potential in the undisturbed state respectively. Since we have changed the definition of the gravitational potential before the relations (2), the equation $\partial\psi_0/\partial\rho=0$ means that the gravitational force is balanced by the centrifugal force. Recall that for this reason the centrifugal force falls out of the characteristic equations (2).

For the disturbed part of the distribution function, denoting it as f, we derive

$$\frac{\partial f}{\partial t}+\frac{1}{\kappa\rho}\frac{\partial\psi}{\partial\varphi}\frac{\partial f_0}{\partial\rho}+\left[\Omega+\frac{v_\perp^2}{4k^2\rho}\frac{d\Omega}{d\rho}\right]\frac{\partial f}{\partial\varphi}+\frac{v_\perp}{4k^2\rho}\frac{d\Omega}{d\rho}\frac{\partial\psi}{\partial\varphi}\frac{\partial f_0}{\partial v_\perp}=0. \tag{7}$$

The Poisson equation for the gravitational potential disturbance ψ is

$$\frac{\partial^2\psi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\psi}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\varphi^2}+\frac{\partial^2\psi}{\partial z^2}=-8\pi^2 GM\int_0^\infty f\,v_\perp\,dv_\perp\,\delta(z), \tag{8}$$

where $\delta(z)$ is Dirac's function, G the Newtonian gravitational constant, and M the mass of the individual star. The expression of the gravitational potential at $z=0$ must be substituted in (7).

The solution of Eq. (8) may easily be found making use (i) of suitable integral transforms of ψ and f and (ii) of the plane wave expansion of Dirac's function. By substituting the expression of the gravitational potential ψ into the solution of Eq. (7) for f, one may reduce the problem to an integral equation, which is a singular homogeneous Fredholm equation of the second kind. But its analysis is rather difficult. For this reason, it is more appropriate to look for local solutions of our problem (Mikhailovsky, 1963; Vedenov *et al.*, 1962; Galeev, 1963). In the case of local solutions, boundary conditions do not enter and k_p is assumed equal to zero $\left(k_\rho\simeq 0,\text{ i.e. }\frac{m}{k_\rho\rho}\gg 1\right)$. The local dispersion relation to be found in this way, permits us to consider the stability problem and to find order of magnitude estimates for the frequencies and the growth rate.

We look for local solutions of Eqs. (7)–(8) in the form

$$f = f(\rho, v_\perp)\, e^{i(-\omega t + m\varphi)}$$
$$\psi = \psi(\rho, z)\, e^{i(-\omega t + m'\varphi)}. \tag{9}$$

Substituting (9) and (10) into (8) and solving it with the aid of Fourier-transforms, we obtain

$$\psi(\rho, z=0) = \frac{4\pi^2 GM}{\frac{|m|}{\rho}} \int_0^\infty f\, v_\perp\, dv_\perp. \tag{10}$$

Then from (7) we derive the dispersion relation

$$1 = \frac{4\pi^2 GM}{\kappa} \operatorname{sgn}(m) \int_0^\infty v_\perp\, dv_\perp \frac{\left(\frac{\partial}{\partial \rho} + \frac{v_\perp}{4\kappa}\frac{d\Omega}{d\rho}\frac{\partial}{\partial v_\perp}\right) f_0}{\omega - m\Omega - \frac{m}{4\kappa^2 \rho}\frac{d\Omega}{d\rho} v_\perp^2}. \tag{11}$$

Eq. (11) is of the type

$$F(\omega_r - m\Omega + i\,\omega_i) = 0.$$

Assuming the overstability to be weak ($d\Omega/d\rho$ to be small), consider ω_i as a small quantity in comparison with the quantity $(\omega_r - m\Omega)$. The calculations below give results which are qualitatively correct only for small ω_i.

Expanding the dispersion relation in powers of ω_i we find

$$F(\omega_r - m\Omega + i\,0) + \frac{\partial F(\omega_r - m\Omega + i\,0)}{\partial(\omega_r - m\Omega)}\, i\,\omega_i + \cdots = 0.$$

If we write

$$F(\omega_r - m\Omega + i\,0) = F_r(\omega_r - m\Omega + i\,0) + i\,F_i(\omega_r - m\Omega + i\,0),$$

where F_r and F_i are both real functions, we find that

$$F_r(\omega_r - m\Omega + i\,0) = 0,$$

and

$$\omega_i = -\frac{F_i(\omega_r - m\Omega + i\,0)}{\frac{\partial F_r(\omega_r - m\Omega + i\,0)}{\partial(\omega_r - m\Omega)}}. \tag{12}$$

If f_0 is of Maxwellian type, i.e.

$$f_0 = \frac{n(\rho)}{\pi\, \overline{v^2}} \exp\left(-\frac{v_\perp^2}{\overline{v^2}}\right),$$

where n is the projected number density, we find

$$F_r = 1 - \chi \int_0^\infty \frac{\frac{dn}{d\rho} - 2\beta\, n\, \xi}{\tilde{\omega} - \gamma\, \xi}\, e^{-\xi}\, d\xi, \tag{13}$$

$$F_i = \frac{\pi\chi}{\gamma} \int_0^\infty \left(\frac{dn}{d\rho} - 2\beta\, n\, \xi\right) e^{-\xi}\, \delta\left(\xi - \frac{\tilde{\omega}}{\gamma}\right) d\xi, \tag{14}$$

where

$$\xi=\frac{v_\perp^2}{\overline{v^2}},\quad \tilde{\omega}=\omega_r-m\,\Omega,\quad \beta=\frac{1}{4\kappa}\frac{d\Omega}{d\rho},\quad \gamma=\frac{m\,\beta\,\overline{v^2}}{\kappa\,\rho},\quad \chi=\frac{2\pi\,GM}{\kappa}\,\mathrm{sgn}(m). \tag{9}$$

It is seen from (13) that the integral is valid if the zero of the denominator is eliminated by a zero numerator, i.e. when $\xi=\frac{\tilde{\omega}\,\kappa\,\rho}{m\,\beta\,\overline{v^2}}$ we must have $\frac{dn}{d\rho}=2n\frac{\tilde{\omega}\,\kappa\,\rho}{m\,\overline{v^2}}$. If $dn/d\rho$ is assumed to be negative, it follows from this fact that $\tilde{\omega}/m<0$. This means that those waves will be overstable whose relative angular velocity has the same direction as the drift motion of the stars. (10)

Since under the integral in (13) the function decreases rapidly as ξ increases only the contribution of small ξ is of importance. Then

$$\begin{aligned} F_r &= 1-\frac{\chi}{\tilde{\omega}}\int_0^\infty\left(\frac{dn}{d\rho}-2\beta\,n\,\xi\right)\left(1+\frac{\gamma}{\tilde{\omega}}\xi\right)\mathrm{e}^{-\xi}\,d\xi \\ &\simeq 1-\frac{\chi}{\tilde{\omega}}\left(\frac{dn}{d\rho}-2\beta\,n+\frac{\gamma}{\tilde{\omega}}\frac{dn}{d\rho}\right) \\ &= 1-\frac{A}{\tilde{\omega}}-\frac{B}{\tilde{\omega}^2}, \end{aligned} \tag{11}$$

where

$$A=\chi\left(\frac{dn}{d\rho}-2\beta\,n\right),\qquad B=\chi\,\gamma\frac{dn}{d\rho}.$$

From the equation $F_r=0$ we have

$$\tilde{\omega}=\frac{A}{2}\pm\left(\frac{A^2}{4}+B\right)^{\frac{1}{2}}.$$

Those $\tilde{\omega}$ must be taken into account here, for which the condition $\tilde{\omega}/m<0$ holds.

Finally

$$\omega_i=-\frac{\pi\,\tilde{\omega}^3}{\gamma}\,\frac{\dfrac{dn}{d\rho}-2\beta\,n\dfrac{\tilde{\omega}}{\gamma}}{(2\gamma+\tilde{\omega})\dfrac{dn}{d\rho}-2\beta\,n\,\tilde{\omega}}\,\mathrm{e}^{-\frac{\tilde{\omega}}{\gamma}}. \tag{12}$$

(15)

As it follows from (15), $\omega_i\to 0$ at $\gamma\to 0$; ω_i will be positive for $m>0$ in the following cases:

$$\left|\frac{dn}{d\rho}\right| > \left|2\beta\,n\frac{\tilde{\omega}}{\gamma}\right|,$$

The ~~first~~ inequality corresponds to a ~~positive~~ negative numerator and a ~~negative~~ positive denominator* and the second one to the reverse situation. The remaining gap in the negative $dn/d\rho$ region is connected with the approximation used, as it ~~is~~ well known in plasma physics (GALEEV *et al.*, 1963b). Thus the magnitude of the density gradient conditions the strength of the overstability. Finally we note once more that both $\tilde{\omega}$ and ω_i vanish when the n and Ω gradients vanish. Hence, in fact, this type of waves is specific for inhomogeneous systems. (13) (14)

*(of the second fraction)

The expression for $\tilde{\omega}$ shows that drift waves are hydrodynamically stable, in the degree of accuracy we have used.

The order of magnitude of $\tilde{\omega}$ may be estimated as

$$\tilde{\omega} \simeq \frac{m}{4k^2\rho}\frac{d\Omega}{d\rho}\overline{v^2} \sim \frac{\overline{v^2}}{4\Omega^2}\frac{\Omega}{4\rho^2} = \frac{1}{4}\frac{r^2}{\rho^2}\Omega,$$

where $r = (\overline{v^2})^{\frac{1}{2}}/2\Omega$. If ρ is of the order of few r, then $\tilde{\omega} \simeq 0.1\,\Omega$, or one order less. The corresponding growth rate is one or two orders less than the frequency, i.e. $\omega_i \simeq (10^{-2}\text{–}10^{-3})\,\Omega$. These estimates characterize the situation for weak gradients only.

III. Concluding Remarks

Drift effects are of great importance for spiral density waves because the inclination angle is finite. Indeed, in the theory of tightly wound spiral arms, the tangent of the inclination angle "i" is regarded as small quantity $\left(\tan i = \frac{m}{k_\rho \rho}\right)$. If "$i$" is small, one may, indeed, neglect drift effects since the ratio of the third term in kinetic equation (1) $\left(\frac{v_\varphi}{\rho}\frac{\partial f}{\partial \varphi}\right)$, which presents the drift motion, to the second one $\left(v_\rho \frac{\partial f}{\partial \rho}\right)$ is just of the order of $\tan i$. It is clear, however, that the inclination angle is finite for real spiral arms. As for those in galaxies of Sc type, this angle is even wide.

A limiting case, opposite to the one considered by LIN and other authors, was discussed above, where $\frac{m}{k_\rho \rho} \gg 1$, or $k_\rho \to 0$. This assumption simplifies the local analysis though it limits the size of the vicinity of any fixed point, say ρ_0, within which the results of the local analysis are valid (MIKHAILOVSKY, 1963; MIKHAILOVSKAYA and MIKHAILOVSKY, 1963; RUKHADZE and SILIN, 1964). It does not present a difficulty, of course, to adopt a step-by-step method in order to take into account the finiteness of the ratio $\frac{m}{k_\rho \rho}$.

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